A-SSE
A LIFETIME OF SAVINGS

PERSONAL FINANCE BIG IDEAS TAUGHT IN THIS TASK:

- **Compound Interest**

TASK

For 70 years, Oseola McCarty earned a living washing and ironing other people's clothing in Hattiesburg, Mississippi. Although she did not earn much money, she budgeted her money wisely, lived within her means, and began saving at a very young age. Before she died, she drew worldwide attention by donating $150,000 to the University of Southern Mississippi for a scholarship fund in her name. The fact that Ms. McCarty was able to save so much money and generously gave it away is an inspiration to many others. She was honored with the Presidential Citizens Medal for her generosity. How did she do it?

Let's assume that she saved the same amount at the end of each year and invested it in a savings account earning 5% per year compounded annually. (When you contribute the same amount each year to an account it is called an *annuity*.) How much do you think Ms. McCarty would have to save each year in order to accumulate $150,000 over a 70-year period?

A. Before we figure it out, take a guess.
   - $100
   - $250
   - $500
   - $1,000
   - $2,000

B. Suppose Ms. McCarty saved $100 and then deposited it at the end of the year in an account that earns 5% interest, compounded annually.
   - How much will it be worth at the end of the second year? At the end of the third year? At the end of the 70th year?
   - Write an expression that represents the value of an investment of *C* dollars after 70 years. Assume as above that it is deposited at the end of the first year in an account that earns 5% interest, compounded annually.

C. Now suppose Ms. McCarty saved another $100 in the *second* year and then deposited it at the end of that year in her account.
   - How much will it be worth at the end of the third year? At the end of the fourth year? At the end of the 70th year?
   - Write an expression that represents the value of an investment of *C* dollars after 69 years.

D. Suppose Ms. McCarty saved $100 *each and every year* for 70 years. Each time, she deposited it in her account at the end of the year.
   - How much would she have saved? What would it be worth at the end of 70 years?
   - Write an expression that represents the value of an investment of *C* dollars deposited each year for 70 years. Assume as above that it is always deposited at the end of the year in an account that earns 5% interest, compounded annually.

E. Had she saved $1,000 a year, how much would she have had after 70 years under the same conditions?

F. How much would she have to save each year in order to accumulate $150,000 after 70 years? How does this compare to your guess? Are your surprised by the answer?

G. The *future value FV* of an annuity is the total value of the annuity after a certain number of years. The formula for the future value of an annuity is shown below.

$$FV = C \cdot \left[\frac{(1+r)^t - 1}{r}\right]$$

   Based on the work you did above, what is the meaning of *C* in this context? What is the meaning of *r* in this context? What is the meaning of *t* in this context?

COMMENTARY

The purpose of this instructional task is to give students an opportunity to construct and find the value of a geometric series (A-SSE.4) in a financial literacy context. The task assumes that students have already developed the formula for a geometric series themselves; having them recognize the need for this formula (and look up if necessary) allows them to engage in MP 5, Use appropriate tools strategically. The task also provides students with an opportunity to look for and express regularity in repeated reasoning (MP 8), as the solution shows. This task also asks students to interpret the variables in the future value formula in the context of the problem (A-SSE.1).

Natural extensions of this task include asking students how much they would have to save each year given different interest rates, numbers of years of saving, and savings goals. Another possible extension is for students to develop the formula for the future value shown in the last part. Alternatively, the work students have done to this point will have prepared them to understand a derivation presented by the teacher.

Students may not know what it means to compound interest, in which case they can be told that interest that is compounded annually is computed at the end of each year and added into the account, thereby increasing the amount of money in the account. For more information about financial literacy concepts that could help students understand this context better, visit, econedlink.

This task is part of a set collaboratively developed by *Money as You Learn*, an initiative inspired by recommendations of the President's Advisory Council on Financial Capability, and Illustrative Mathematics. Integrating essential financial literacy concepts into the teaching of the Common Core State Standards can strengthen teaching of the Common Core and expose students to knowledge and skills they need to become financially capable young adults. A mapping of essential personal finance concepts and skills against the Common Core State Standards as well as additional tasks and texts will be available at www.moneyasyoulearn.org. This task and additional personal finance-related mathematics tasks are available at www.illustrativemathematics.org and are tagged "financial literacy."

SOLUTIONS

A. Answer (b) is the closest to the correct answer, but there is no right or wrong answers here!

B. We note that for this and future parts in this solution, we round computations to the nearest cent.

Year	Value (in dollars)
1	100
2	$100 \cdot (1.05) = 105$
3	$[100 \cdot (1.05)] \cdot 105 = 100 \cdot (1.05)^2 = 110.25$
4	$[100 \cdot (1.05^2)] \cdot 105 = 100 \cdot (1.05)^3 = 115.76$
...	...
70	$100 \cdot (1.05)^{69} = 2897.76$

That is, the investment will be worth \$105 at the end of the second year, \$110.25 at the end of the third year, and \$2,897.76 at the end of the 70th year.

Replacing 100 by C, we see that an expression that represents the value of an investment of C dollars after 70 years is $C \cdot (1.05)^{69}$, or approximately $29C$.

C.

Year	Value (in dollars)
2	100
3	$100 \cdot (1.05) = 105$
4	$[100 \cdot (1.05)] \cdot 105 = 100 \cdot (1.05)^2 = 110.25$
5	$[100 \cdot (1.05^2)] \cdot 105 = 100 \cdot (1.05)^3 = 115.76$
...	...
70	$100 \cdot (1.05)^{68} = 2759.77$

That is, \$100 deposited at the end of the second year will be worth \$105 at the end of the third year, \$110.25 at the end of the fourth year, and \$2,759.77 at the end of the 70th year.

An expression that represents the value of an investment of C dollars after 69 years is $C \cdot (1.05)^{68}$, or approximately $27.6C$.

D. Ms. McCarty saved $100 \cdot 70$ or \$7,000 over the 70 years. The technique developed above shows us how to find the ending value of each of these deposits:

- **The \$100 invested in year 1 is worth $100 \cdot (1.05)^{69}$ at the end of the 70 years.**
- **The \$100 invested in year 2 is worth $100 \cdot (1.05)^{68}$ at the end of the 70 years.**
- **The \$100 invested in year 3 is worth $100 \cdot (1.05)^{67}$ at the end of the 70 years.**
- **…and so on, until …**

- **The $100 invested in year 69 is worth $100 \cdot (1.05)^1$ at the end of the 70 years.**
- **The $100 invested in year 70 is worth $100 \cdot (1.05)^0$ at the end of the 70 years.**
  Now we just need to need to add these up: We recognize the sum

$$100 \cdot (1.05)^{69} + 100 \cdot (1.05)^{68} + 100 \cdot (1.05)^{67} + \cdots + 100 \cdot (1.05)^1 + 100 \cdot (1.05)^0$$

as a geometric series, and so we can evaluate it as

$$100 \cdot \left[\frac{(1.05)^{70} - 1}{1.05 - 1}\right] = 58852.85$$

We conclude that she would have accumulated $58,852.85 all together by setting aside $100 each year for 70 years. Note that this is $\frac{55{,}852.85}{7{,}000} \approx 8.4$ times as much as the money she saved.

An expression that represents the value of an investment of *C* dollars deposited each year for 70 years is

$$C \cdot \left[\frac{(1.05)^{70} - 1}{.05}\right] = 588.5C$$

E. We can use the last part of the problem above to find this:

$$1000 \cdot \left[\frac{(1.05)^{70} - 1}{.05}\right] = 588528.51$$

Of course, we could also note that she is saving 10 times as much each year so the total she saves will be ten times as great. In either case we see that she would have saved $588,528.51 all together.

F. We can use what we have already found:

$$C \cdot \left[\frac{(1.05)^{70} - 1}{.05}\right] = 150000$$

$$C \cdot 588.53 = 150000$$

$$C = \frac{150000}{588.53}$$

So she would need to save approximately $254.87 each year to accumulate $150,000 in 70 years. Whether this is surprising depends on the student and their initial guess.

G. Based on the work above, *C* is the amount of money added to the annuity every year, *r* is the annual interest rate (expressed as a decimal), and *t* is the number of years the same amount of money is deposited in the annuity.